

Spatial economics for granular settings


Jonathan I. Dingel (Chicago Booth)

Felix Tintelnot (UChicago)

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Local economic shocks at fine spatial resolution

Local economic shocks. . .

- Workplace employment/productivity
e.g., Amazon's proposed HQ2 in New York City
- Land/housing/residential amenities
e.g., Detroit's neighborhood revitalization projects
- Transportation costs
e.g., Berlin's new U5 connection of underground lines 

. . . at fine spatial resolution (Rosenthal and Strange, 2020)

- Arzaghi and Henderson (2008): productivity gains from interactions within 500 meters
- Rossi-Hansberg, Sarte and Owens (2010): housing externalities halve every 1,000 feet
- Ahlfeldt et al. (2015): production & residential externalities halve within 1-2 minutes

Quantitative spatial models in granular settings

- Spatial linkages (commuting, trade, local externalities, etc) govern the incidence of local economic shocks
- Want “an empirically relevant quantitative model to perform general equilibrium counterfactual policy exercises” (Redding and Rossi-Hansberg, 2017)
- Continuum of agents \rightarrow realized shares = model probabilities [► Literature](#)
- Consider a granular setting. Two concerns arise when the number of spatial links is large relative to the number of decision makers:
 1. Risk of overfitting the model to the idiosyncratic components of individual decisions
 2. Counterfactual outcomes may be sensitive to the idiosyncratic components of individual decisions

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Spatial economics for granular settings: Roadmap

Computing counterfactuals in continuum models

Counterfactual analysis in granular empirical settings

- Applying continuum model to NYC 2010

- Monte Carlo: Calibrated-shares procedure overfits data

- Event studies: Neighborhood employment booms

A spatial model with a finite number of individuals

Application to Amazon's HQ2

Computing counterfactual outcomes in continuum models

Continuum model: Economic environment

- Each location has productivity A and land endowment T
- Measure L individuals w/ one unit of labor and hired by competitive firms producing freely traded goods differentiated by location of production
- Individuals have Cobb-Douglas preferences over goods $(1 - \alpha)$ and land (α)
- Commuting costs: $\delta_{kn} = \underbrace{\bar{\delta}_{kn}}_{\text{time}} \times \underbrace{\lambda_{kn}}_{\text{disutility}}$
- Individuals have idiosyncratic tastes for pairs of residential and workplace locations, such that i 's utility from living in k and working in n is

$$U_{kn}^i = \epsilon \ln \left(\frac{w_n}{r_k^\alpha P^{1-\alpha} \delta_{kn}} \right) + \nu_{kn}^i \quad \nu_{kn}^i \stackrel{\text{iid}}{\sim} \text{T1EV} \quad (1)$$

Continuum model: Equilibrium

Given economic primitives $(\alpha, \epsilon, \sigma, L, \{A_n\}, \{T_k\}, \{\delta_{kn}\})$, an equilibrium is a set of wages $\{w_n\}$, rents $\{r_k\}$, and labor allocation $\{\ell_{kn}\}$ such that

$$\text{labor allocation (gravity):} \quad \frac{\ell_{kn}}{L} = \frac{w_n^\epsilon (r_k^\alpha \delta_{kn})^{-\epsilon}}{\sum_{k', n'} w_{n'}^\epsilon (r_{k'}^\alpha \delta_{k'n'})^{-\epsilon}} \quad \forall k, n \quad (2)$$

$$\text{goods markets:} \quad A_n \sum_k \frac{\ell_{kn}}{\bar{\delta}_{kn}} = \frac{(w_n/A_n)^{-\sigma}}{P^{1-\sigma}} Y \quad \forall n \quad (3)$$

$$\text{land markets:} \quad T_k = \frac{\alpha}{r_k} \sum_n \underbrace{\frac{\ell_{kn}}{\bar{\delta}_{kn}} w_n}_{y_{kn}} \quad \forall k, n \quad (4)$$

$$\left(\frac{1+\epsilon}{\sigma+\epsilon}\right) \left(\frac{\alpha\epsilon}{1+\alpha\epsilon}\right) \leq \frac{1}{2} \implies \text{unique equilibrium (Allen, Arkolakis and Li, 2023)}$$

Continuum model: Counterfactual outcomes

Define $\hat{x} \equiv \frac{x'}{x}$. Counterfactual equilibrium system can be expressed as

$$\hat{w}_n = \hat{A}_n \left(\sum_k \hat{y}_{kn} \frac{y_{kn}}{\sum_{k'} y_{k'n}} \right)^{\frac{1}{1-\sigma}} \left(\sum_{n'} \left(\frac{\hat{w}_{n'}}{\hat{A}_{n'}} \right)^{1-\sigma} \sum_k \frac{y_{kn'}}{Y} \right)^{\frac{1}{1-\sigma}} \hat{Y}^{\frac{1}{\sigma-1}} \quad (5)$$

$$\hat{r}_k = \hat{T}_k^{-1} \sum_n \hat{y}_{kn} \frac{y_{kn}}{\sum_{n'} y_{kn'}} \quad (6)$$

$$\hat{\ell}_{kn} = \begin{cases} 1, & \text{if } \ell_{kn} = 0 \\ \frac{\hat{w}_n^\epsilon \left(\hat{r}_k^\alpha \hat{\delta}_{kn} \hat{\lambda}_{kn} \right)^{-\epsilon}}{\sum_{k',n'} \hat{w}_{n'}^\epsilon \left(\hat{r}_{k'}^\alpha \hat{\delta}_{k'n'} \hat{\lambda}_{k'n'} \right)^{-\epsilon} \frac{\ell_{k'n'}}{L}} & \text{if } \ell_{kn} > 0 \end{cases} \quad (7)$$

“Exact hat algebra”: Compute \hat{w}_n , \hat{r}_k , and $\hat{\ell}_{kn}$ given elasticities σ , α , and ϵ , baseline shares $\frac{\ell_{kn}}{L}$ and $\frac{y_{kn}}{Y}$, and relative exogenous parameters \hat{A}_n , \hat{T}_k , $\hat{\delta}_{kn}$ and $\hat{\lambda}_{kn}$.

Continuum model: Fitting the model to data

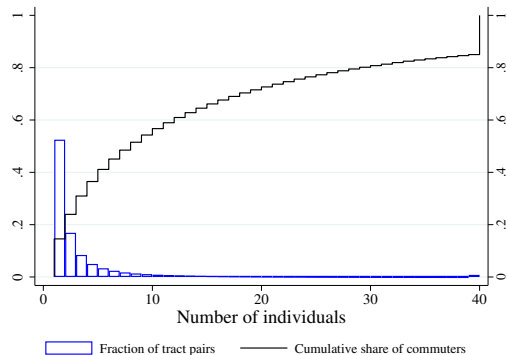
- In general, we distinguish defining a system of equations to solve for counterfactual equilibria from fitting a model's parameters
- Exact hat algebra concerns comparative statics, not calibration
- “Calibrated shares” (often used interchangeably with “exact hat algebra”)
 - Dominant approach to counterfactual analysis in quantitative spatial models uses **observed** shares in equations (5)-(7)
 - Implicitly calibrates parameters so model exactly delivers the observed shares (e.g., $\ell_{kn} = 0 \implies \delta_{kn} = \infty$)
- Covariates-based approach (e.g., Ahlfeldt et al. 2015)
 - Parameterize δ_{kn} as function of observed covariates
 - Use **fitted** model's values of the baseline shares in equations (5)-(7)
- As we will discuss, many alternatives lie between these two approaches

Counterfactual analysis in granular empirical settings

Commuting flows in granular settings

NYC has 2.5 million resident-employees and 4.6 million tract pairs.

- 85% of tract pairs have zero commuters between them
- 41.1% of commuters in cell with ≤ 5
- 44% of NYC tract pairs with positive flow in 2013 were zeros in 2014
- Gravity model predicts 2014 value better than 2013 value for bottom 95% of tract pairs (▶)



Source: Longitudinal Employer-Household Dynamics, Origin Destination Employment Statistics. LODS employment counts are noise-infused and LODS flows are synthetically generated.

▶ Detroit

▶ MSP

▶ Counties

▶ Impersistent: Detroit

▶ Imp: counties

▶ Asymmetric zeros

▶ State-to-state migration

Contrasting parameterizations of commuting costs

- Pick $\alpha = 0.24$, $\sigma = 4$, $L =$ number of employed individuals
- Seek values of $\{\delta_{kn}\}$, ϵ , $\{T_k\}$, $\{A_n\}$

$$\delta_{kn} = \underbrace{\bar{\delta}_{kn}}_{\text{observed}} \times \underbrace{\lambda_{kn}}_{\text{unobserved}}$$

- Compute $\{\bar{\delta}_{kn}\}$ from Google Maps transit times: $\bar{\delta}_{kn} = \frac{H}{H - t_{kn} - t_{nk}}$
 1. Covariates-based approach:
Assume $\lambda_{kn} = 1 \ \forall k, n$
 2. Calibrated-shares procedure:
Assume structural error λ_{kn} appropriately orthogonal

Estimating the commuting elasticity

Covariates-based: Logit log likelihood function

(McFadden, 1974, 1978; Guimarães, Figueiredo and Woodward, 2003)

$$\ln \mathcal{L} = \sum_k \sum_n \ell_{kn} \ln \left[\frac{w_n^\epsilon (r_k^\alpha \bar{\delta}_{kn})^{-\epsilon}}{\sum_{k',n'} w_{n'}^\epsilon (r_{k'}^\alpha \bar{\delta}_{k'n'})^{-\epsilon}} \right]$$

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Calibrated shares: Commuting gravity eqn

$$\frac{\ell_{kn}}{L} = \frac{w_n^\epsilon (r_k^\alpha \bar{\delta}_{kn} \lambda_{kn})^{-\epsilon}}{\sum_{k',n'} w_{n'}^\epsilon (r_{k'}^\alpha \bar{\delta}_{k'n'} \lambda_{k'n'})^{-\epsilon}}$$

$\mathbb{E}(\lambda_{kn}^{-\epsilon} | \cdot) = 1 \rightarrow$ estimate ϵ by PPMLE (Silva and Tenreyro, 2006)

Estimating the commuting elasticity for NYC in 2010

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	NYC (2010) PPML/MLE
Commuting cost	-7.986 (0.307)
Model fit (pseudo- R^2)	0.662
Location pairs	4,628,878
Commuters	2,488,905

NOTES: Specification includes residence fixed effects and workplace fixed effects.

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Covariates-based approach: Solve for $\{T_k\}$ and $\{A_n\}$ using fixed effects ($\propto r_k^{-\alpha\epsilon}$ and w_n^ϵ) and equations (2), (3), and (4)

Calibrated-shares procedure: Use estimated ϵ

► Interactive fixed effects

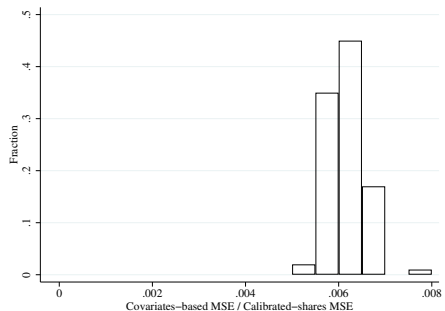
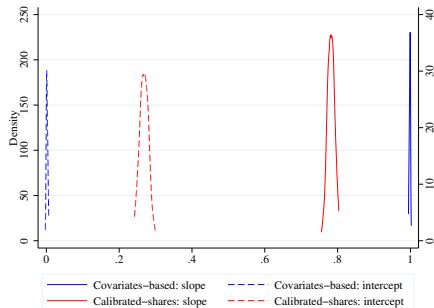
Monte Carlo: Applying each procedure to finite data

- DGP is estimated covariates-based model for NYC in 2010
- Simulated “event”: \uparrow productivity of 200 Fifth Ave tract by 9%
- Apply calibrated-shares procedure and covariates-based approach
(Increase A_n to match total employment increase in simulated data)
- Does the procedure predict the change in the number of commuters from each residential tract working in the “treated” tract?
- Regress “true” changes on predicted changes (2160 obs per simulation)
Ideally, want slope = 1 and intercept = 0
- Compute forecast errors (MSE for “true” vs predicted changes)

Monte Carlo: Calibrated-shares procedure overfits

Apply each procedure to simulated “2010” data. 100 simulations w/ $I = 2,488,905$

Changes in commuter counts ($\ell'_{k\bar{n}} - \ell_{k\bar{n}}$)



I	2.5	5	12.5	25	50	125	250	2560
Calibrated-shares: slope	0.782	0.876	0.948	0.974	0.986	0.995	0.997	1.000
Calibrated-shares: intercept	0.269	0.153	0.064	0.032	0.017	0.007	0.004	0.000
Calibrated-shares: MSE	0.225	0.113	0.045	0.023	0.011	0.005	0.002	0.000

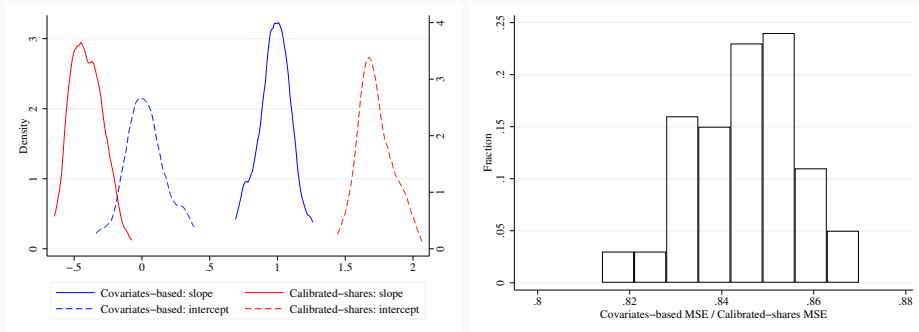
► Changes in rents

► Λ

Monte Carlo: Calibrated-shares procedure overfits

Apply each procedure to simulated “2010” data. 100 simulations w/ $I = 2,488,905$

Changes in commuter counts ($\ell'_{k\bar{n}} - \ell_{k\bar{n}}$) via finite-sample draws from pre- and post- DGPs



I	2.5	5	12.5	25	50	125	250	2560
Calibrated-shares: slope	-0.408	0.194	0.669	0.835	0.913	0.968	0.982	0.998
Calibrated-shares: intercept	1.724	0.982	0.404	0.202	0.106	0.040	0.022	0.002
Calibrated-shares: MSE	17.022	8.486	3.400	1.699	0.851	0.340	0.169	0.017

Using tract-level events to evaluate model performance

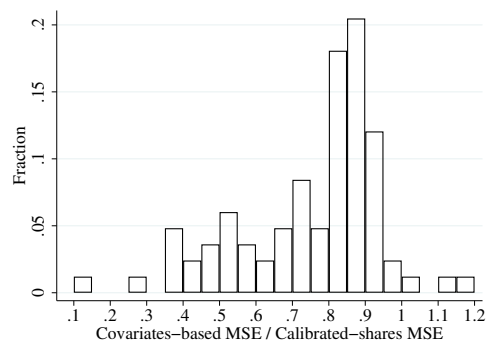
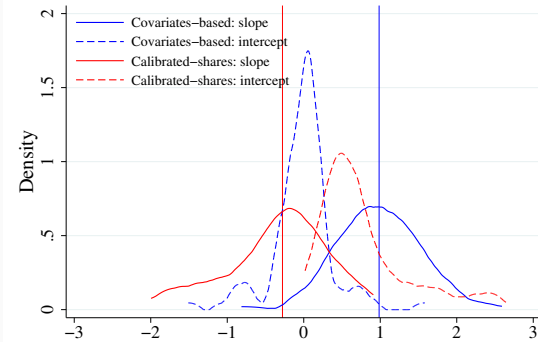
Kehoe (2005): “it is the responsibility of modelers to demonstrate that their models are capable of predicting observed changes, at least ex post”

How well do models predict changes in commuting flows?

- Look at 83 tract-level employment booms ($\geq +12.5\%$) in NYC in 2010–2012
- We raise productivities in tracts to match observed changes in total employment
- Does the model predict changes in bilateral commuting flows to that destination? (n.b. total employment change need not be exogenous)
 - Regress observed changes on predicted changes
 - Contrast forecast errors (MSEs)

Comparison of models' predictive performance across 83 events

Covariates-based model much better at predicting change in number of commuters from each residential tract to booming workplace tract



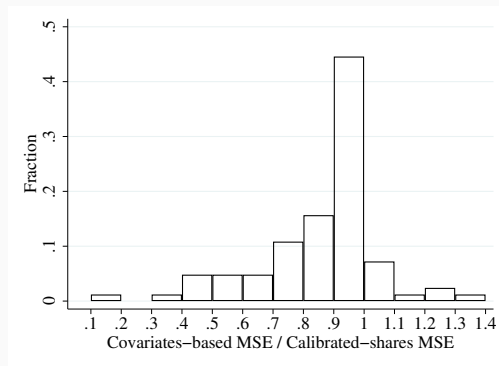
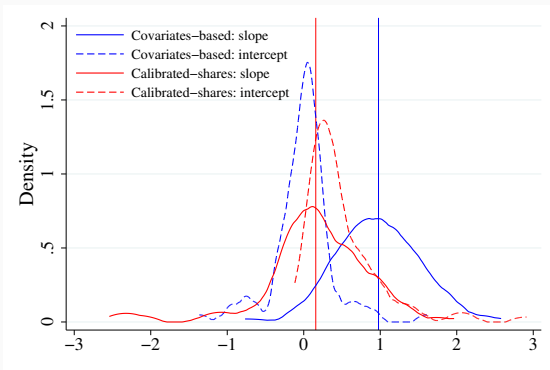
► No extensive margin

Comparisons with alternative approaches

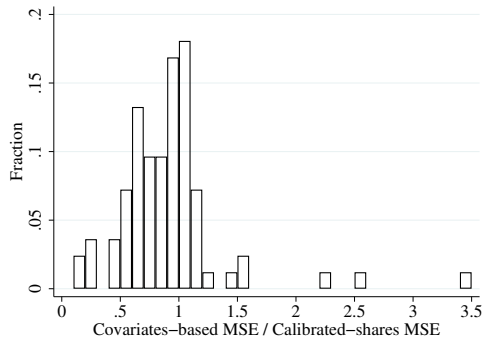
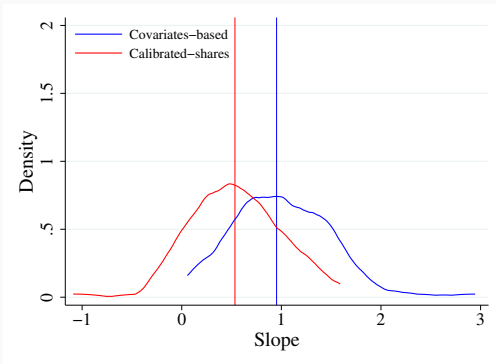
We compare the performance of the covariates-based specification to other parameterization methods, including

- using **pooled** pre-event data for 2008-2010,
- aggregating counterfactual predictions to the Neighborhood Tabulation Area (**NTA**) level,
- using a low-rank approximation of the commuting matrix computed with singular value decomposition (**SVD**), and
- using fitted values from an enriched covariates-based model including **interactive fixed effects**.

Comparisons over 83 events with pooled data



Comparisons over 83 events at NTA level



► Estimation at NTA level

Calibrated “fitted” shares: SVD

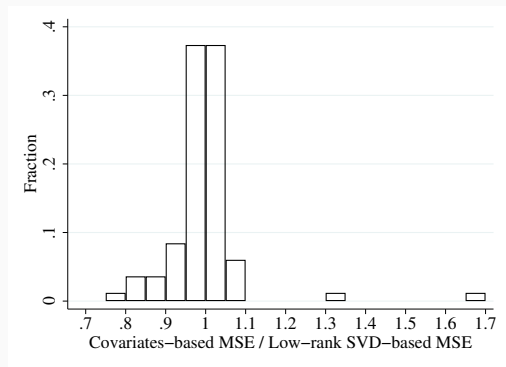
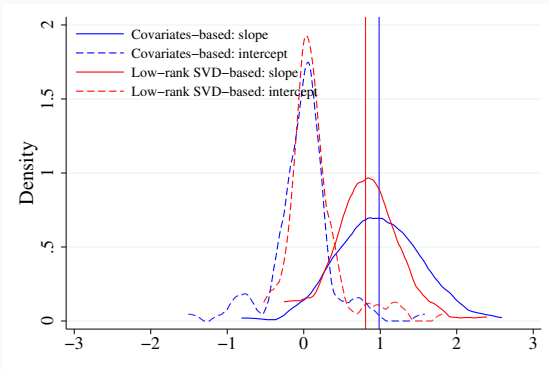
For this exercise, we replace the commuting matrix with a low-rank approximation.

- For the commuting matrix $\mathbf{L} = [\ell_{kn}]$ and fixed rank \mathfrak{r} , solve

$$\min_{\tilde{\mathbf{L}}} \left| \tilde{\mathbf{L}} - \mathbf{L} \right| \quad \text{s.t.} \quad \text{rank}(\tilde{\mathbf{L}}) \leq \mathfrak{r}$$

- SVD factors \mathbf{L} as USV' , where U and V are orthonormal and S is diagonal and non-negative, with \mathbf{L} 's singular values as entries.
- By keeping the largest \mathfrak{r} values in S and setting the rest to zero, we obtain the optimal rank \mathfrak{r} approximation, per the Eckart-Young theorem.
- We replace all negative values with zeros and rescale so that $\sum \ell_{kn} = \sum \tilde{\mathbf{L}}_{kn}$, and use the observed wages from 2010.

Calibrated “fitted” shares: SVD, rank 16



► Choosing rank

► Alternative ranks

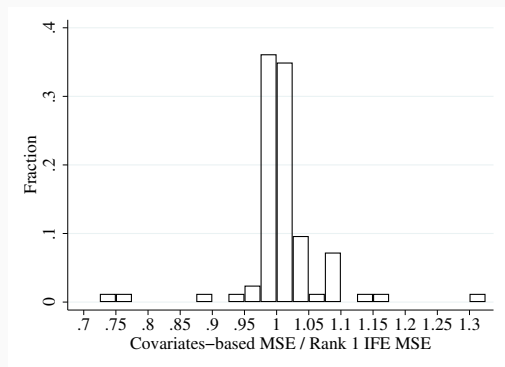
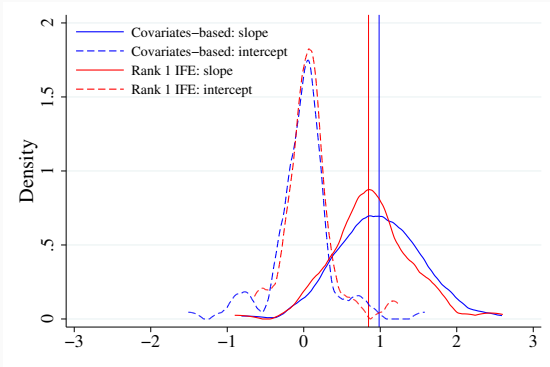
► Matrix visualizations

Calibrated “fitted” shares: Interactive fixed effects

Interactive fixed effects is a generalization of the covariates-based model and represents a midpoint between the covariates-based and calibrated-observed-shares.

- For the covariates-based specification, we assumed that there were no unobserved commuting costs $\lambda_{kn} = 1 \ \forall kn$.
- Now let $\lambda_{kn} = \exp(\psi'_k \gamma_n)$, with ψ_k and γ_n both $R \times 1$ vectors.
 R is the rank of the implied factor structure
- Estimate ψ and γ , residence FEs, workplace FEs, and commuting elasticity ϵ by maximum likelihood.

Calibrated “fitted” shares: Rank 1 interactive fixed effects



Counterfactuals in continuum models: Takeaways

We examined varied strategies for estimating/calibrating the model of baseline shares

- Calibrating millions of parameters using millions of observed shares has severe overfitting problem
- Time aggregation (pooling 3 years) is insufficient
- Parsimonious transit-time parameterization performs well in event studies
- SVD (of ranks 6 to 16) performs similarly
- More flexible interactive-fixed-effect specification offers modest improvement

**A spatial model with a finite
number of individuals**

A spatial model with a finite number of individuals

Goal: examine the sensitivity of counterfactual outcomes to the idiosyncratic component of individual decisions

In the limit ($I \rightarrow \infty$), the equilibrium of our model with an integer number of individuals is (almost surely) the equilibrium of the continuum model

Modeling concerns raised by the integer number of individuals:

- Individuals must have beliefs about equilibrium wages and land prices

$$\binom{I + N^2 - 1}{N^2 - 1} = \frac{(I + N^2 - 1)!}{(N^2 - 1)!I!} \quad I = 10, N = 4 \implies 3.27 \times 10^6$$

- There will be a *distribution* of equilibria for each set of parameters Υ

Model: Economic environment

- Each location has productivity A and land endowment T
- I individuals are endowed with L/I units of labor and hired by competitive firms producing freely traded goods differentiated by location of production
- Individuals have Cobb-Douglas preferences over goods and land
- Individuals have idiosyncratic tastes for residence-workplace pairs
- Workers know primitives $\Upsilon \equiv (L, \{A_n\}, \{T_k\}, \{\bar{\delta}_{kn}\}, \{\lambda_{kn}\}, \alpha, \epsilon, \sigma)$ and have (common) point-mass beliefs \tilde{r}_k and \tilde{w}_n about land prices and wages
- Worker i knows own idiosyncratic preferences $\{\nu_{kn}^i\}$ but not the full set of idiosyncratic residence-workplace draws ν^I

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Timing: Individuals choose labor allocation, then markets clear

1. Workers choose the kn pair that maximizes

$$\tilde{U}_{kn}^i = \epsilon \ln \left(\frac{\tilde{w}_n}{\tilde{P}^{1-\alpha} \tilde{r}_k^\alpha \delta_{kn}} \right) + \nu_{kn}^i$$

given point-mass beliefs \tilde{r}_k and \tilde{w}_n

2. After choosing kn based on their beliefs, workers are immobile and cannot relocate
3. Given the labor allocation $\{\ell_{kn}\}$ and economic primitives Υ , a **trade equilibrium** is a set of wages $\{w_n\}$ and land prices $\{r_k\}$ that clears all markets.

Commuting equilibrium with a finite number of individuals

Given primitives Υ , idiosyncratic residence-workplace draws $\boldsymbol{\nu}^I$, and point-mass beliefs $\{\tilde{w}_n\}, \{\tilde{r}_k\}$, a **commuting equilibrium with a finite number of individuals**, I , is defined as a labor allocation $\{\ell_{kn}\}$, wages $\{w_n\}$, and land prices $\{r_k\}$ such that

- $\ell_{kn} = \frac{L}{I} \sum_{i=1}^I \mathbf{1}\{\tilde{U}_{kn}^i(\boldsymbol{\nu}^I) > \tilde{U}_{k'n'}^i(\boldsymbol{\nu}^I) \ \forall (k', n') \neq (k, n)\}$; and
- wages $\{w_n\}$ and land prices $\{r_k\}$ are a *trade equilibrium* given the labor allocation $\{\ell_{kn}\}$.

Convergence to the continuum model equilibrium

- Definition: Given primitives $\Upsilon \equiv (L, \{A_n\}, \{T_k\}, \{\bar{\delta}_{kn}\}, \{\lambda_{kn}\}, \alpha, \epsilon, \sigma)$, \tilde{w} and \tilde{r} are “**continuum-case rational expectations**” if \tilde{w} and \tilde{r} constitute a trade equilibrium for the labor allocation $\{\ell_{kn}\}$ given by equation (2).
- Result: As $I \rightarrow \infty$, if individuals’ point-mass beliefs are continuum-case rational expectations, then the equilibrium quantities and prices of the model with a finite number of individuals coincide (almost surely) with those of the continuum model.

Estimating the finite model

Likelihood (McFadden, 1974, 1978; Guimarães, Figueiredo and Woodward, 2003)

$$\ln \mathcal{L} = \sum_k \sum_n \ell_{kn} \ln \left[\frac{\tilde{w}_n^\epsilon (\tilde{r}_k^\alpha \bar{\delta}_{kn})^{-\epsilon}}{\sum_{k',n'} \tilde{w}_{n'}^\epsilon (\tilde{r}_{k'}^\alpha \bar{\delta}_{k'n'})^{-\epsilon}} \right]$$

- Solve for $\{T_k\}$ and $\{A_n\}$ using fixed effects ($\propto \tilde{r}_k^{-\alpha\epsilon}$ and \tilde{w}_n^ϵ) under continuum-case rational expectations
- This estimation procedure yields same ϵ , $\{T_k\}$, and $\{A_n\}$ as the covariates-based continuum model

Ex post regret is small

- Individuals make residence-workplace choices based on wage and rent beliefs
- The realized equilibrium wages and rents will differ ► Price dispersion
- Calculate ex post regret χ_i at realized prices for i who chose kn :

$$\max_{k', n'} \left(\epsilon \ln \left(\frac{w_{n'}}{P^{1-\alpha} r_{k'}^\alpha \delta_{k'n'}} \right) + \nu_{k'n'}^i \right) = \left(\epsilon \ln \left(\frac{(1 + \chi_i) w_n}{P^{1-\alpha} r_k^\alpha \delta_{kn}} \right) + \nu_{kn}^i \right)$$

- Quantitatively modest: 96% would not want to switch ► Switchers
- Conditional on wanting to switch, median ex-post regret χ_i is 0.7%.

► Price dispersion in ex post regret simulations

Comparison with continuum model

- Idiosyncratic residence-workplace draws $\nu^I \rightarrow$ distributions of equilibrium quantities and prices (for given primitives Υ)
- Mean equilibrium outcomes:
 - Mean commuter counts coincide with those from the continuum model
$$\frac{\ell_{kn}}{L} = \mathbb{E} \left[\Pr(U_{kn}^i > U_{k'n'}^i \ \forall (k', n') \neq (k, n)) \right]$$
 - Land prices and wages are solved from a non-linear system of equations
- Variance of equilibrium outcomes due to idiosyncrasies
 - Confidence interval for residents, workers, wages, and prices
- In counterfactual exercises: Change from Υ to Υ' for given ν^I

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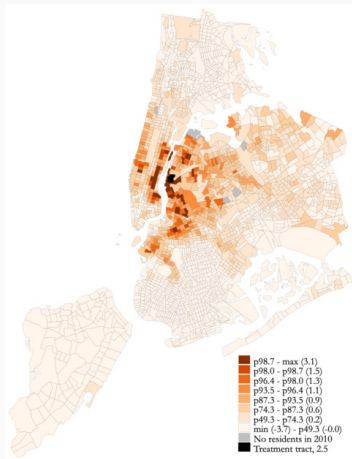
Application to Amazon's HQ2

Counterfactual: Amazon HQ2 in Long Island City

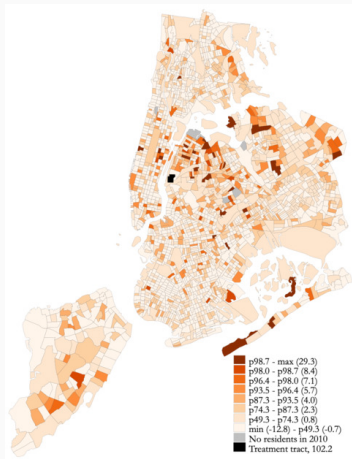
- Amazon's 2017 RFP for HQ2 with 50,000 employees elicited 238 proposals
- NYC proposed four possible sites (and controversial tax breaks)
- Split siting announced in 2018 would have put 25,000 employees in Long Island City
- Quantitative questions: What would happen to NYC neighborhoods with this local employment boom? Are these changes large relative to uncertainty stemming from idiosyncratic component of individuals' decisions?

Contrasting predictions for changes in residents

Calibrated-shares predictions are spatially idiosyncratic



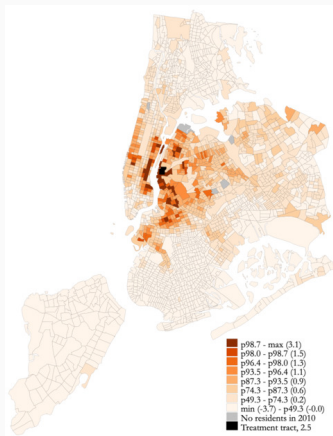
Covariates-based model



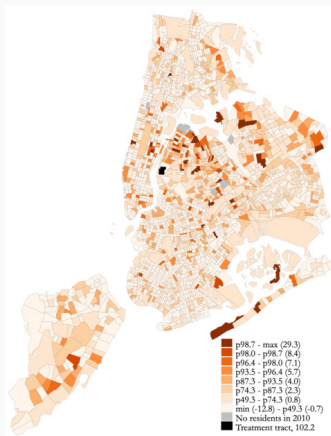
Calibrated-shares procedure

Contrasting predictions for changes in residents

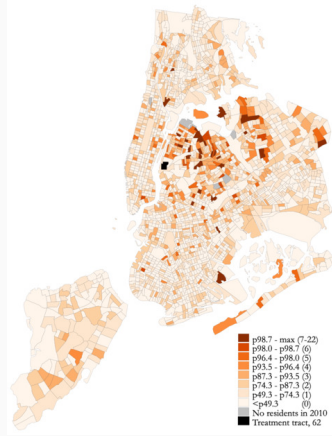
Calibrated-shares predictions are tightly tied to initial residents ► Workers



Covariates-based model

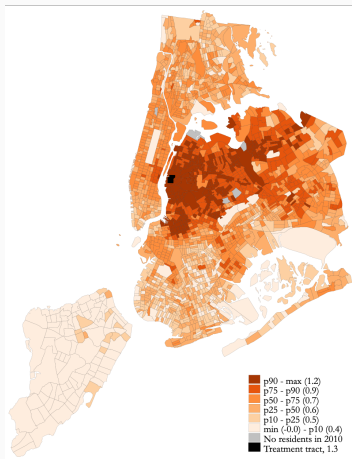


Calibrated-shares procedure

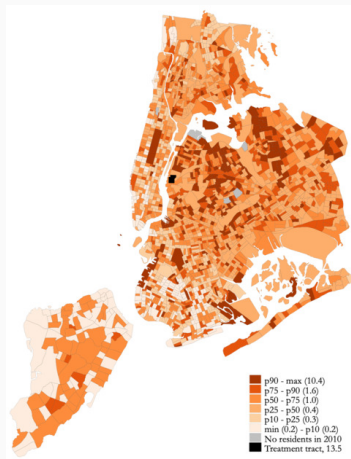


Residents working at AHQ2 tract

Contrasting predictions for changes in rents



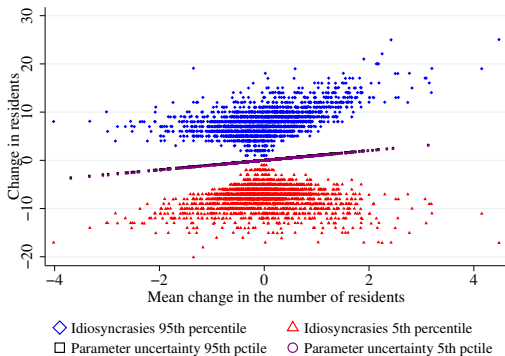
Covariates-based model



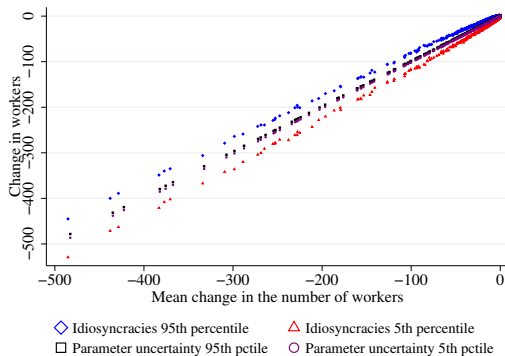
Calibrated-shares procedure

Sizable uncertainty about predicted changes from idiosyncrasies

Changes in residents

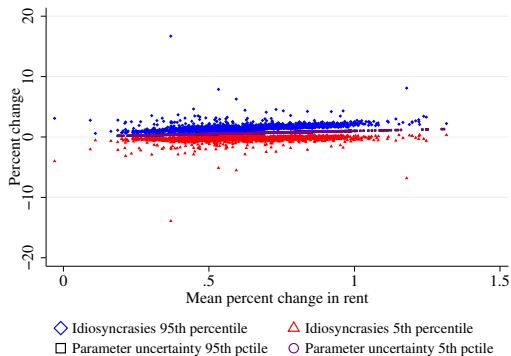


Changes in workers

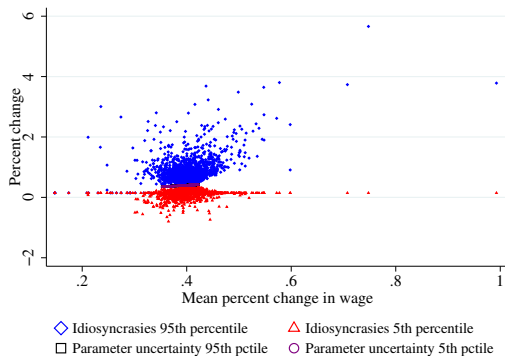


Sizable uncertainty about predicted changes from idiosyncrasies

Changes in rents



Changes in wages



Conclusions

Conclusions

- Finer spatial data are exciting but not a free lunch
- We need to evaluate the performance of applied GE models
- Monte Carlo and event studies: Calibrated-shares procedure performs poorly in granular empirical settings
- Parsimonious covariates-based specification predicts quite well
- New tools: use fitted shares (e.g., low-rank matrix approximations) rather than observed shares in exact hat algebra
- Uncertainty about counterfactual predictions induced by individual idiosyncrasies can be sizable

Thank you

Predicting the incidence of local economic shocks

Workplace employment: “The new 15-year lease agreement with property owner L&L Holding Co. will allow Tiffany to unite employees at the company’s three headquarters locations under one roof. Formerly known as the International Toy Center, the approximately 800,000-square-foot building at 200 Fifth recently emerged from a massive makeover at the hands of L&L.” (*CP Executive*, 30 Apr 2010)

Residential amenities: “A recent report by the Urban Institute warns of ‘green gentrification,’ where public investment in green spaces – like the 606 trail –can raise property values, attract development and wealthier residents, and price existing residents out of the area.” (*Chicago Reporter*, 30 Jan 2020)

Transportation costs: “After completion, the U5 gap closure will give the major residential areas in the east of Berlin a direct connection to the historic city centre, the government district and the central station. . . Once the U5 gap has been closed, 20 percent of private vehicle traffic is expected to shift to the new U5.” (projekt-u5.de)

Many applications infer infinite costs from zeros

- Heblich, Redding and Sturm (2020): “For all pairs of boroughs with zero commuting flows, our model implies prohibitive commuting costs, and we make this assumption to ensure that the model is consistent with the observed data.”
- Monte, Redding and Rossi-Hansberg (2018): “model implies prohibitive commuting costs for pairs with zero commuting flows” and “the model implies prohibitive trade costs for pairs with zero trade”
- Severen (2021): “most pairs that are ever zero (in either 1990 or 2000) are always zero. Always zero pairs do not contribute any variation to models with pair fixed effects”

Gravity-based estimates better predict 2014 commuter counts

# of commuters	Share	Gravity: time	2013 values	Gravity: distance	2013 values
Panel A: Detroit					
≤ 5	0.960	0.384	0.308	0.367	0.307
≤ 10	0.983	0.494	0.473	0.465	0.472
Panel B: NYC					
≤ 5	0.978	0.362	0.306	0.373	0.306
≤ 10	0.990	0.474	0.475	0.477	0.473



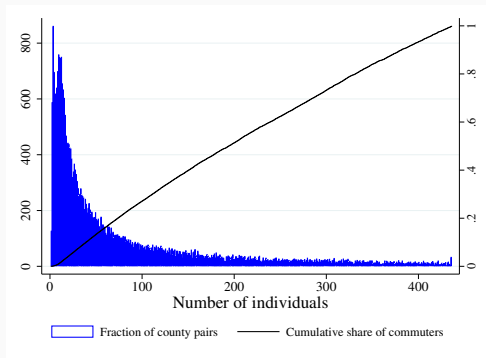
ACS state-to-state migration matrix is s.t. sampling noise

2001	2002						
	0	1	2	3	4	5	6+
0	0.59	0.16	0.13	0.05	0.03	0.01	0.02
1	0.40	0.17	0.17	0.10	0.06	0.05	0.06
2	0.32	0.16	0.21	0.09	0.07	0.07	0.09
3	0.23	0.16	0.21	0.09	0.08	0.09	0.14
4	0.26	0.13	0.15	0.10	0.06	0.06	0.25
5	0.13	0.09	0.12	0.14	0.14	0.06	0.31
6+	0.03	0.03	0.06	0.06	0.06	0.07	0.69

- The ACS 2001 dataset has 644,427 prime-age individuals out of the total sample of 1,192,206 individuals.
- Of 73,101 individuals who moved residences, 80.6% migrated within their states and 19.4% (14,215) moved between-states.
- Migration flows are winsorized at 6 and values are given as a percentage of 2001 migration flows, so rows sum to one.

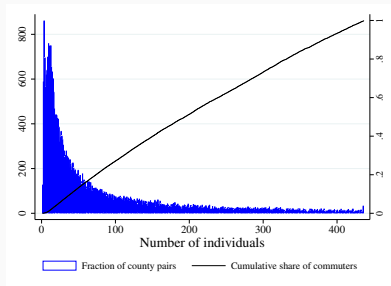
Some counties migration flows are s.t. sampling noise

- 35 million cross-county commuters between 79,188 pairs of counties within 120km (MRR 2018) in 2006–2010 American Community Survey
- Skewed: For the bottom 90% of pairs, the mean value is only 40 commuters
- 45% of county pairs within 120 km have zero commuters between them.



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 - Skewed: For the bottom 90% of pairs, the mean value is only 40 commuters
 - 45% of county pairs within 120 km have zero commuters between them.
 - ACS is a 1-in-20 representative sample
-
- 55% of county pairs with a positive number of commuters represent ~ 5 or fewer respondents (≤ 100 commuters)
 - 34% of county pairs with a positive number of commuters report a number of commuters that is less than the Census-reported margin of error.



Zeros are asymmetric, but daily commutes are roundtrip journeys

When $\ell_{kn} > 0$, we often observe $\ell_{nk} = 0$:

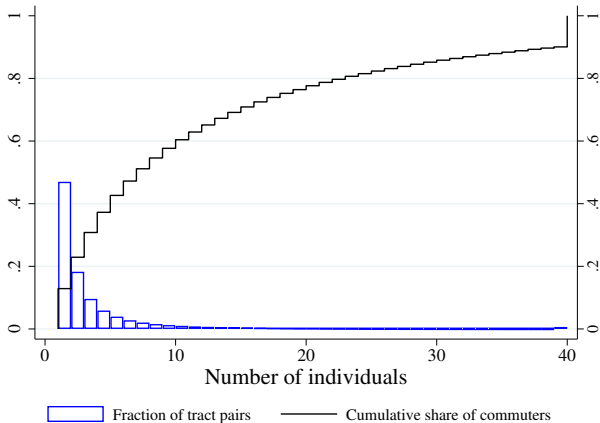
- US counties: $\ell_{nk} = 0$ for 22% of county pairs with $\ell_{kn} > 0$.
- Detroit tracts: $\ell_{nk} = 0$ for 66% of tract pairs with $\ell_{kn} > 0$.
- Brazilian municipios: $\ell_{nk} = 0$ for 49% of municipio pairs with $\ell_{kn} > 0$.

If infinite commuting costs rationalize $\ell_{nk} = 0$, how do you go from k to n in morning and return from n to k in evening?

- Commuting costs must switch between finite and infinite within each day
- Zero commuters cannot make congestion a source of intra-day variation

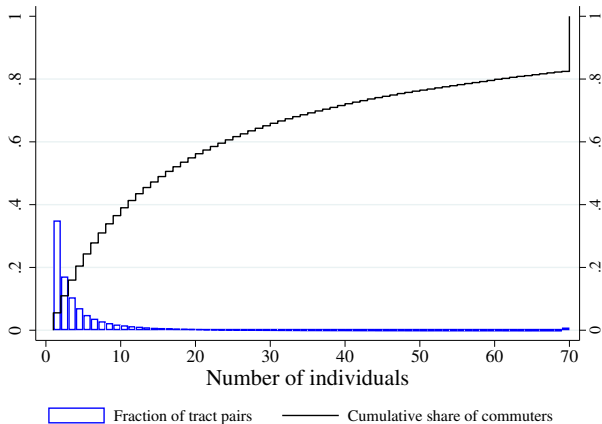
Statistics for Detroit

- The number of tract pairs and number of commuters are both about 1.3 million.
- 42.6% of commuters in cell with ≤ 5
- 74% of tract pairs have zero commuters between them
- $\ell_{nk} = 0$ for 66% of tract pairs with $\ell_{kn} > 0$.



Statistics for Minneapolis-St Paul

- LODES data for Minnesota is reported by establishment (rather than firm)
- Zeros are pervasive: 61%
- Zeros are asymmetric:
 $\ell_{nk} = 0$ for 54% of tract pairs with $\ell_{kn} > 0$.



Commuter counts are impersistent: Detroit

First symptom of finite noise:

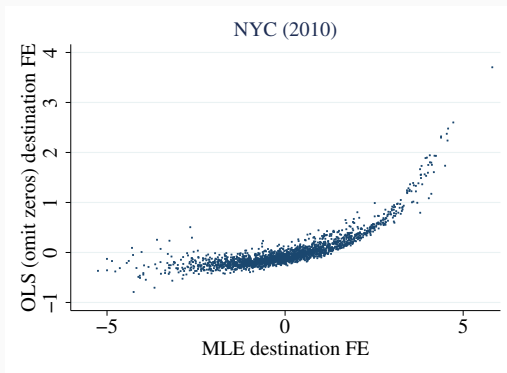
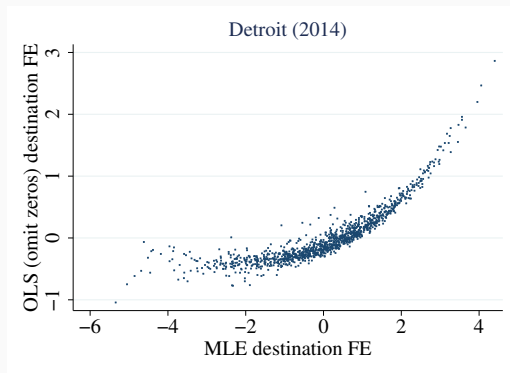
- Little mass on transition matrix's diagonal
- 39% of Detroit tract pairs with positive flow in 2013 were zeros in 2014
- Gravity model predicts 2014 value better than 2013 value for bottom 95% of tract pairs ➡

2013	2014					
	0	1	2	3	4	5+
0	0.86	0.10	0.02	0.01	0.00	0.00
1	0.60	0.22	0.10	0.04	0.02	0.02
2	0.37	0.25	0.16	0.09	0.06	0.08
3	0.23	0.22	0.18	0.13	0.08	0.16
4	0.15	0.17	0.17	0.14	0.11	0.26
5+	0.04	0.06	0.07	0.08	0.08	0.68

Commuter counts are impersistent: US counties

		2011–2015								
Initial Share(%)		0	1–30	31–50	51–70	71–90	91–110	111–500	501–1,500	>1,500
2006–2010	0	45.88	0.78	0.18	0.02	0.01	0.00	0.00	0.00	0.00
	1–30	19.64	0.35	0.46	0.10	0.05	0.02	0.01	0.02	0.00
	31–50	5.47	0.16	0.36	0.19	0.12	0.07	0.04	0.07	0.00
	51–70	3.42	0.08	0.26	0.18	0.15	0.10	0.08	0.15	0.00
	71–90	2.50	0.05	0.16	0.15	0.16	0.14	0.12	0.23	0.00
	91–110	1.86	0.02	0.11	0.12	0.15	0.12	0.13	0.35	0.00
	111–500	12.02	0.00	0.02	0.03	0.04	0.05	0.05	0.74	0.07
	501–1,500	5.01	0.00	0.00	0.00	0.00	0.00	0.00	0.13	0.81
>1,500	4.07	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	
										0.94

Fixed effect estimates are biased by dropping zeros



$\mathbb{E}(\ell_{kn} | \ell_{kn} > 0) \geq \mathbb{E}(\ell_{kn}) \rightarrow$ popular procedure attributes lower employment counts to infinite commuting costs, not lower wages/productivity

Interactive fixed effects specification

We can parameterize the unobserved commuting costs λ_{kn} as $\exp(\psi'_k \gamma_n)$, where ψ_k and γ_n are $R \times 1$ vectors.

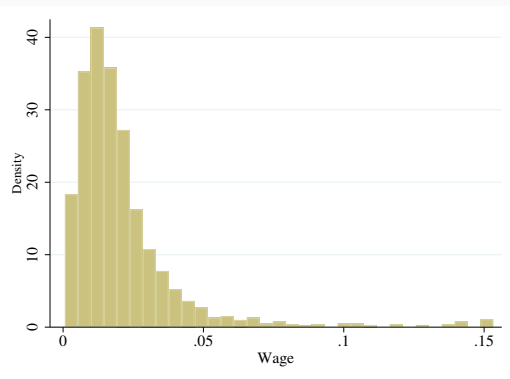
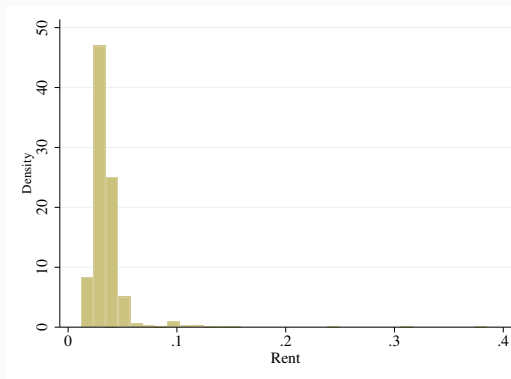
- The dimensions of ψ and γ determine the rank of the implied factor structure.
 - As R increases, the computational difficulty of estimating the resulting model increases rapidly.
- For the covariates-based model estimation with only origin and destination fixed effects, we write $R = 0$.

Estimation: Interactive fixed effects

Commuting elasticity estimates for NYC 2010, varying R

	$R = 0$	$R = 1$	$R = 2$	$R = 3$	$R = 4$	
$\hat{\epsilon}$	-7.9842	-7.1762	-6.6521	-6.3586	-5.7359	
pseudo- R^2	0.662	0.684	0.694	0.701	0.706	◀ Back
Location pairs			4,628,880			
Commuters			2,488,905			

Price dispersion across finite-model equilibria



NOTES: The plots depict the dispersion of prices (r_k/P or w_n/P) for each tract in New York City using the finite model estimated on 2010 data. Left panel depicts dispersion in tracts' rents, which have a median value of 0.032 (p5 = 0.021, p95 = 0.051). Right panel depicts dispersion in tracts' wages, which have a median value of 0.016 (p5 = 0.004, p95 = 0.054).

Ex post regret in the finite model

<i>s</i>	Share with regret	Unconditional distribution					Conditional distribution	
		p95	p96	p97	p98	p99	Mean	Median
1	0.0442	0.0000	0.0011	0.0042	0.0082	0.0150	0.0106	0.0073
2	0.0433	0.0000	0.0009	0.0039	0.0078	0.0143	0.0102	0.0071
3	0.0446	0.0000	0.0012	0.0043	0.0083	0.0150	0.0106	0.0072
4	0.0446	0.0000	0.0012	0.0043	0.0084	0.0152	0.0106	0.0073
5	0.0437	0.0000	0.0010	0.0040	0.0079	0.0144	0.0103	0.0071
6	0.0444	0.0000	0.0012	0.0042	0.0083	0.0150	0.0107	0.0073
7	0.0447	0.0000	0.0013	0.0043	0.0083	0.0150	0.0105	0.0072
8	0.0445	0.0000	0.0012	0.0043	0.0084	0.0150	0.0106	0.0073
9	0.0452	0.0000	0.0014	0.0045	0.0086	0.0154	0.0109	0.0074
10	0.0444	0.0000	0.0011	0.0042	0.0082	0.0148	0.0106	0.0072
mean	0.0444	0.0000	0.0012	0.0042	0.0083	0.0149	0.0106	0.0072

NOTES: The table reports the share of individuals with ex post regret and the utility gains of their desired switches in simulations of our estimated finite model. The first column identifies the simulation s . The second column reports the fraction of individuals who have ex post regret and therefore would prefer a different choice given realized prices. Columns under "Unconditional distribution" report the distribution of utility gain based on full sample ($I = 2,488,905$). Columns under "Conditional distribution" report the distribution of utility gain among those who would want to switch. The "mean" row reports the mean value across ten simulations.

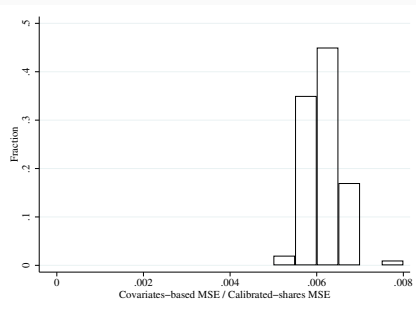
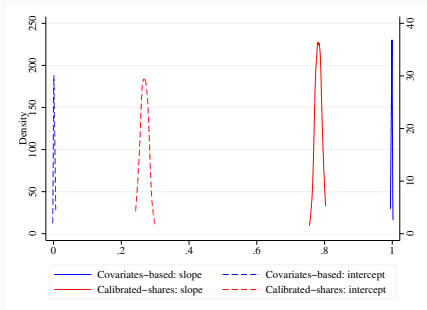
Price dispersion across finite-model equilibria

Simulation count	mean	p5	p10	p25	p50	75	p90	p95
Wage								
100,000	0.021	0.004	0.006	0.010	0.016	0.025	0.039	0.054
10	0.023	0.004	0.005	0.009	0.015	0.025	0.039	0.057
Rent								
100,000	0.035	0.021	0.024	0.027	0.032	0.038	0.045	0.051
10	0.034	0.016	0.019	0.024	0.031	0.038	0.049	0.058

NOTES: This table compares the price (r_k/P or w_n/P) dispersion generated by the simulations of the finite model (100,000 simulations) and the ex post regret calculations (10 simulations).

Monte Carlo: Calibrated-shares procedure overfits

Apply each procedure to simulated “2010” data. 100 simulations w/ $I = 2,488,905$
Changes in rents (\hat{r}_k / \hat{P})



I	2.5	5	12.5	25	50	125	250	2560
Calibrated-shares: slope	0.192	0.311	0.537	0.696	0.820	0.918	0.958	0.996
Calibrated-shares: intercept	0.808	0.689	0.464	0.304	0.180	0.082	0.042	0.004
Calibrated-shares: MSE	417.560	225.459	85.328	43.469	21.960	9.332	4.884	1.049

► $\hat{\ell}_{kn}$ and \hat{r}_k

Monte Carlo: Calibrated-shares procedure overfits

λ	I	Covariates-based	Calibrated-shares	Covariates-based	Calibrated-shares
0	2.5	0.9985	0.7817	0.0014	0.2252
0	5	0.9992	0.8759	0.0007	0.1130
0	12.5	0.9995	0.9479	0.0003	0.0452
0	25	0.9998	0.9737	0.0001	0.0227
0	50	0.9999	0.9864	0.0001	0.0112
0	125	1.0000	0.9946	0.0000	0.0045
0	250	1.0000	0.9971	0.0000	0.0023
0	2560	1.0000	0.9997	0.0000	0.0002
0.1	2.5	1.0005	0.7901	0.0371	0.2254
0.1	5	1.0013	0.8818	0.0364	0.1136
0.1	12.5	1.0020	0.9480	0.0360	0.0448
0.1	25	1.0022	0.9748	0.0359	0.0226
0.1	50	1.0022	0.9867	0.0358	0.0113
0.1	125	1.0023	0.9951	0.0358	0.0045
0.1	250	1.0023	0.9971	0.0358	0.0023
0.1	2560	1.0023	0.9997	0.0358	0.0002
0.25	2.5	1.0033	0.8227	0.2328	0.2264
0.25	5	1.0044	0.9021	0.2321	0.1127
0.25	12.5	1.0045	0.9581	0.2318	0.0452
0.25	25	1.0047	0.9788	0.2316	0.0226
0.25	50	1.0049	0.9895	0.2315	0.0113
0.25	125	1.0049	0.9958	0.2315	0.0045
0.25	250	1.0049	0.9979	0.2315	0.0023
0.25	2560	1.0049	0.9997	0.2315	0.0002
0.5	2.5	1.0048	0.8907	1.0513	0.2263
0.5	5	1.0056	0.9416	1.0504	0.1122
0.5	12.5	1.0062	0.9764	1.0501	0.0450
0.5	25	1.0063	0.9876	1.0498	0.0226
0.5	50	1.0065	0.9932	1.0497	0.0113
0.5	125	1.0066	0.9976	1.0497	0.0045
0.5	250	1.0066	0.9990	1.0497	0.0022
0.5	2560	1.0066	0.9999	1.0497	0.0002
1	2.5	0.9954	0.9688	6.3762	0.2176
1	5	0.9965	0.9837	6.3749	0.1092
1	12.5	0.9972	0.9933	6.3745	0.0441
1	25	0.9969	0.9965	6.3750	0.0218
1	50	0.9971	0.9982	6.3748	0.0109
1	125	0.9971	0.9994	6.3747	0.0044
1	250	0.9972	0.9995	6.3746	0.0022
1	2560	0.9972	0.9998	6.3746	0.0002

Relationship between $\hat{\ell}_{kn}$ and \hat{r}_k

The compensating variation Ψ is

$$\Psi = \frac{1}{\hat{\ell}_{kn}} \hat{w}_n^\epsilon \left(\hat{r}_k^\alpha \hat{P}^{1-\alpha} \hat{\delta}_{kn} \hat{\lambda}_{kn} \right)^{-\epsilon}, \forall k, n.$$

Taking logarithms on both sides yields the log-log linear relationship between changes in commuter counts ($\hat{\ell}_{k\bar{n}}$) and changes in rents (\hat{r}_k)

$$\log(\hat{\ell}_{k\bar{n}}) = -\alpha\epsilon \log(\hat{r}_k) + C,$$

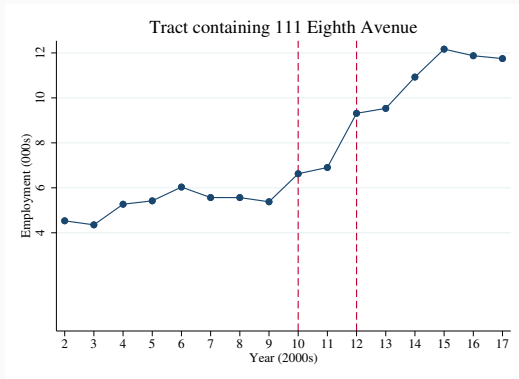
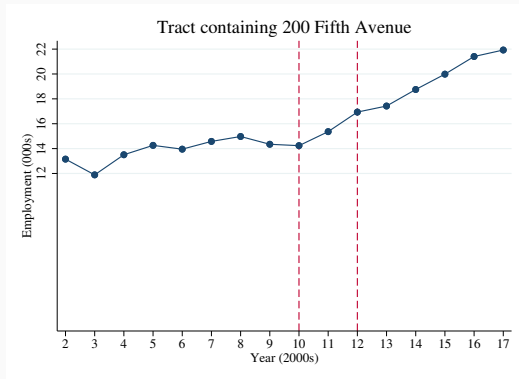
where $C = \epsilon \log(\hat{w}_n) + (1 - \alpha) \log(\hat{P}) - \epsilon \log(\hat{\delta}_{kn}) - \epsilon \log(\hat{\lambda}_{kn}) - \log(\Psi)$.

Monte Carlo DGP with unobserved λ_{kn}

Λ	I	Slope (mean)		MSE (mean)	
		Covariates-based	Calibrated-shares	Covariates-based	Calibrated-shares
0	2.5	0.9796	-0.4075	14.3836	17.0222
0.1	2.5	1.0013	-0.3439	14.3945	16.9868
0.25	2.5	1.0056	-0.1295	14.7522	17.1357
0.5	2.5	1.0154	0.3214	15.4545	17.0197
1	2.5	0.9897	0.8044	20.7317	16.7998

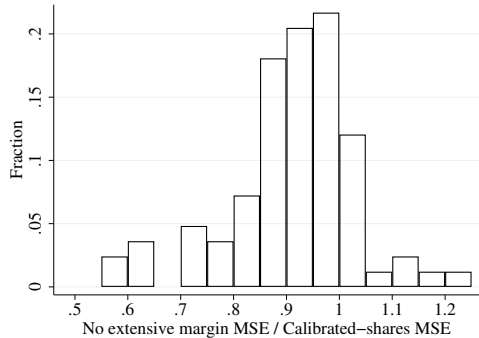
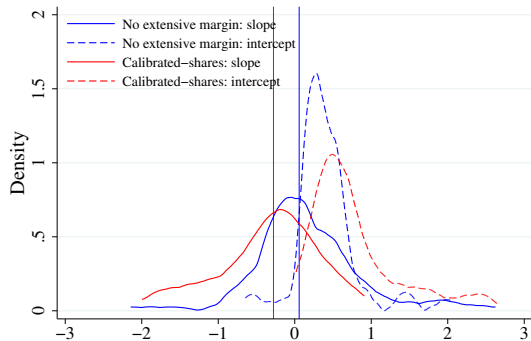
Λ is the variance of λ_{kn} relative to variance of δ_{kn}

Employment increases in the anchor-tenant tracts

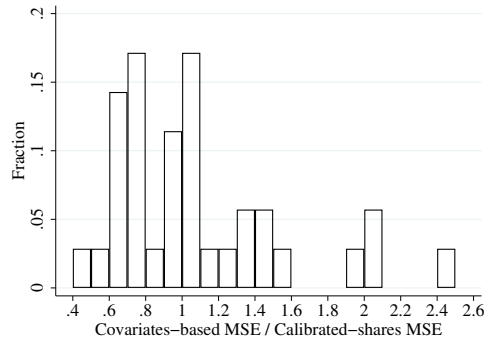
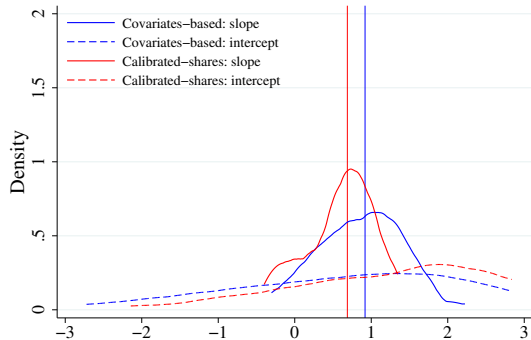


NOTES: This figure depicts the number of primary jobs in tracts 36061005800 and 36061008300 in the LODS data.

Comparisons over 83 events with no extensive margin

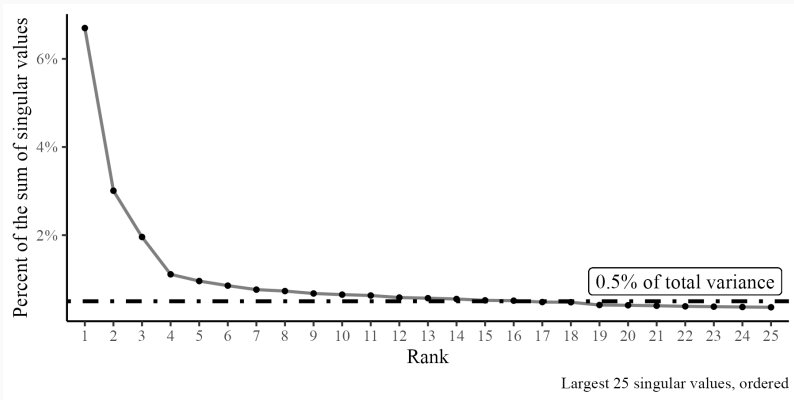


Comparisons over 35 events with NTA-level model



Scree plot for NYC 2010 commuting matrix

Explanatory share of ordered singular values:

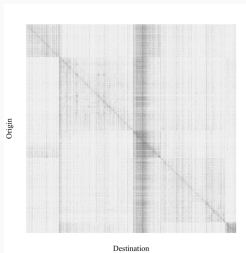


Prediction performance with alternative SVD ranks

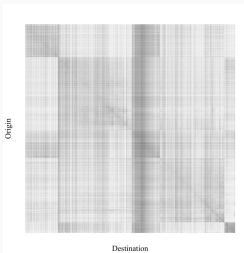
	Rank																			
	1	2	3	4	5	6	8	10	12	14	15	16	18	20	50	100	500	1000	1500	2143
<i>Monte Carlo performance</i>																				
Slope	1.03	1.05	1.05	1.04	1.02	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.99	.91	.78	.61	.78	.78	.78
Int.	-.039	-.060	-.057	-.049	-.021	-.014	-.002	-.005	-.005	-.004	-.003	-.001	.002	.011	.110	.265	.268	.268	.269	.269
MSE	.1583	.0462	.0461	.0460	.0370	.0357	.0331	.0320	.0321	.0316	.0313	.0309	.0305	.0305	.0893	.2214	.5637	.2252	.2252	.2252
<i>Event study performance</i>																				
Slope	.73	.70	.71	.80	.83	.86	.85	.83	.83	.82	.82	.81	.80	.79	.62	.32	-.43	-.47	-.47	-.46
Int.	.06	.14	.14	.09	.08	.08	.10	.13	.13	.14	.14	.15	.16	.17	.30	.51	.80	.82	.82	.82
MSE	10.53	10.38	10.37	10.29	10.27	10.26	10.27	10.30	10.32	10.32	10.38	10.40	10.45	10.48	10.96	11.71	12.94	13.23	13.35	13.39

[◀ Back](#)

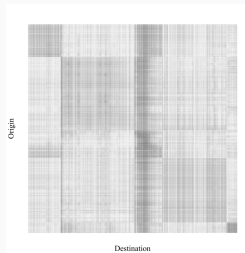
Visualizations of commuting matrices



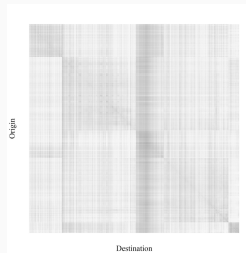
2010 LODS



Covariates-based

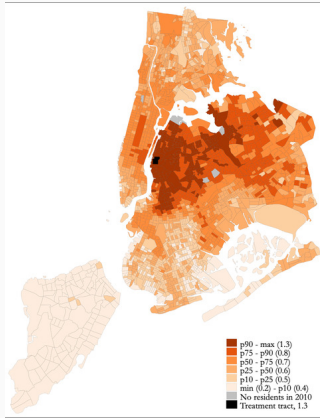


SVD rank 16

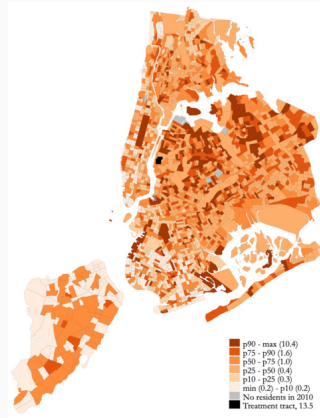


IFE rank 1

Contrasting predictions for changes in land prices

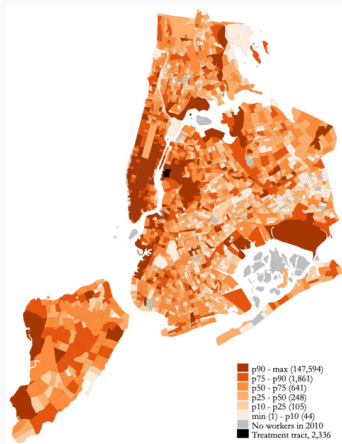


Covariates-based model

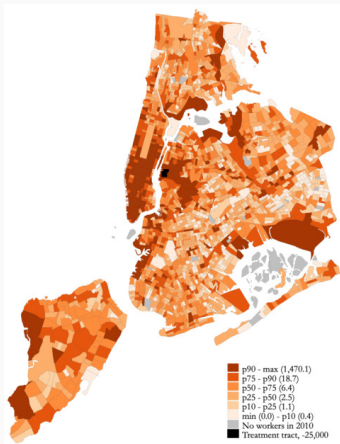


Calibrated-shares procedure

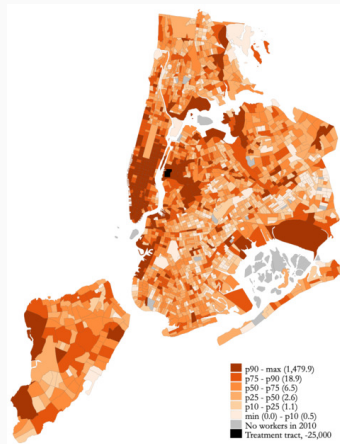
Predictions for changes in workers



Number of workers

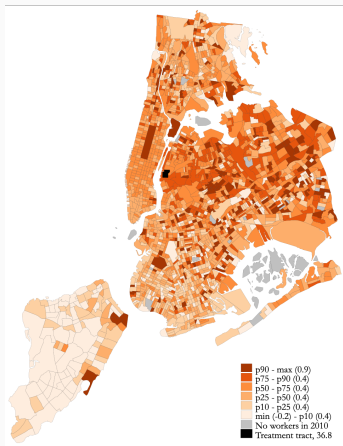


Covariates-based model

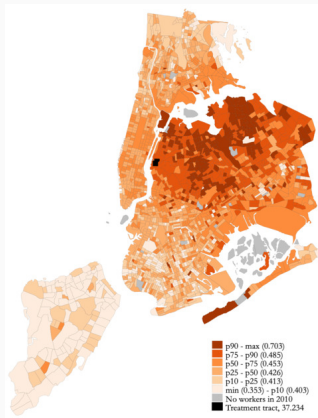


Calibrated-shares procedure

Contrasting predictions for changes in wages



Covariates-based model



Calibrated-shares procedure